Projective bundles

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Dedicated to J. A. Thas on his fiftieth birthday

Abstract

A projective bundle in PG$(2, q)$ is a collection of $q^2 + q + 1$ conics that mutually intersect in a single point and hence form another projective plane of order q. The purpose of this paper is to investigate the possibility of partitioning the $q^5 - q^2$ conics of PG$(2, q)$ into $q^2(q - 1)$ disjoint projective bundles. As a by-product we obtain a multiplier theorem for perfect difference sets that generalizes a portion of Hall's theorem.

1 Introduction

There are $q^5 - q^2 = q^2(q - 1)(q^2 + q + 1)$ nondegenerate conics in the desarguesian projective plane π_0 =PG$(2, q)$ of order q [6, p. 140]. Moreover, it is not hard to find (see [1, §8], [5, p. 1085], or [8]) a collection of $q^2 + q + 1$ nondenegerate conics in π_0 that mutually intersect in exactly one point, and hence serve as the "lines" of another projective plane on the points of π_0.

We will call such a collection of conics a *projective bundle*. The issue of concern for this paper is whether the $q^5 - q^2$ conics of π_0 can be partitioned into $q^2(q - 1)$ projective bundles. We exhibit a collection of $q^2(q - 1)/2$ disjoint bundles for any odd prime power q, and show that a slightly larger number of disjoint bundles may be constructed for $q = 3$. When q is even, a similar construction produces only $q - 1$ disjoint bundles, although a computer-aided search for $q = 4$ produced 30 disjoint bundles. It seems unlikely, however, that a complete partitioning of the conics of π_0 into projective bundles is possible. We also discuss the connections of this problem

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