LOGIC AND p-RECOGNIZABLE SETS OF INTEGERS

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Abstract

We survey the properties of sets of integers recognizable by automata when they are written in p-ary expansions. We focus on Cobham's theorem which characterizes the sets recognizable in different bases p and on its generalization to $\mathbb{N}^m$ due to Semenov. We detail the remarkable proof recently given by Muchnik for the theorem of Cobham-Semenov, the original proof being published in Russian.

1 Introduction

This paper is a survey on the remarkable theorem of A. Cobham stating that the only sets of numbers recognizable by automata, independently of the base of representation, are those which are ultimately periodic. The proof given by Cobham, even if it is elementary, is rather difficult [15]. In his book [24], S. Eilenberg proposed as a challenge to find a more reasonable proof. Since this date, some researchers found more comprehensible proofs for subsets of $\mathbb{N}$, and more generally of $\mathbb{N}^m$. The more recent works demonstrate the power of first-order logic in the study of recognizable sets of numbers [54, 49, 50].

One aim of this paper is to collect, from Büchi to Muchnik's works [9, 54], all the base-dependence properties of sets of numbers recognizable by finite automata, with some emphasis on logical arguments. It contains several examples and some logical proofs. In particular, the fascinating proof recently given by A. Muchnik is detailed

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