

# SURVEY

## On several aspects of $J$ -inner functions in Schur analysis

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### Abstract

The aim of this paper is to give a presentation of several subjects of Schur analysis with some historical information. The class of  $J$ -inner functions plays a key role in this new mathematical field which is situated at the seam of various mathematical disciplines (operator theory, scattering theory, complex function theory, prediction theory for stochastic processes, spectral theory for differential operators). This article shows the importance of  $J$ -inner functions in Schur analysis. We shall concentrate on the Soviet roots of the theory and discuss Potapov's factorization theory and Arov's investigations on Darlington synthesis. Furthermore we present some of Arov's results on interrelations between a certain subclass of  $J$ -inner functions and generalized bitangential Schur-Nevanlinna-Pick interpolation.

### 0 Introduction

In the last 25 years one could observe an intensive research interest in matricial and operator-theoretical versions of classical moment problems and related questions in interpolation theory. This was mainly initiated by the fundamental papers of V.M. Adamjan, D.Z. Arov and M.G. Kreĭn [AAK68a]–[AAK71b]. In the first 30 years of our century, the original scalar versions of these problems attracted

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