Transversal spreads

Norman L. Johnson

Abstract

Knarr shows that given a derivable affine plane, every line not in the associated derivable net produces a spread which is a dual spread in some $PG(3, K)$, for K a skewfield. More generally, if a derivable net has a transversal T, there is also a spread in $PG(3, K)$. This article generalizes results of Knarr by an investigation of spreads in three-dimensional projective spaces realized as transversals to derivable nets. As an application of the ideas, the finite derivable affine planes which are 'partially flag-transitive' are determined.

1 Introduction.

The author's work on derivable nets shows that every derivable net is combinatorially equivalent to a three-dimensional projective space over a skewfield K. More precisely, the points and lines of the net become the lines and points skew to a fixed line N. Recently, Knarr [18] proved that, for any derivable affine plane π, every line ℓ not belonging to the derivable net embeds to a set of lines in the projective space such this set union N becomes a spread $S(\ell)$ of $PG(3, K)$ which is also a dual spread.

Hence, it is of interest to ask what sorts of spreads arise from a given derivable affine plane. We first point out that it is not actually the existence of the derivable plane which provides the spread nor that of the stronger condition that there is an affine plane containing a derivable net but simply the existence of a 'transversal' to a derivable net. That is, given a transversal to a derivable net, there is a corresponding spread of the projective space associated combinatorially or 'geometrically' by the embedding process. The nature of the transversal determines whether the spread constructed is also a dual spread.

Received by the editors December 2000.

Communicated by J. Thas.

1991 *Mathematics Subject Classification :* Primary 51 E 23, Secondary 51 A 40.

Key words and phrases : transversal, spreads, derivable nets.