Further radii in topological algebras

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Abstract

We introduce two new radii in general topological algebras. The first one, η, plays a role similar to that of the norm in Banach algebras in the sense that the series $\sum x^n$ converges whenever $\eta(x) < 1$. The second one permits, among others, to give new expressions of the spectral radius ρ and the boundedness radius β in a non-commutative locally m-convex algebra. Finally, we show that, in contrast to the locally convex setting, β need not be dominated by ρ in a topological (even F-) algebra with continuous inversion.

1 Introduction

In a Banach algebra $(A, ||\ ||)$, the series $\sum x^n := \sum_{n=1}^{\infty} x^n$ converges in A whenever $||x|| < 1$ and its limit is nothing but $-x^o$, x^o being the quasi-inverse of x in A. Actually, this is also true [7] in any normed algebra whose set of quasi-invertible elements is open, i.e. which is a Q-algebra in the sense of I. Kaplanski [6]. In some non-normed topological algebras, the spectral radius ρ still plays the role of the norm in the sense that, if $\rho(x) < 1$, then the series above converges. In some other algebras, it is the boundedness radius β which plays this role. However, there exist topological algebras with elements x such that the series diverges although $\rho(x) < 1$ or $\beta(x) < 1$. In section 2, we introduce a new radius in any topological algebra, called radius of ńig-boundedness and denoted by η, in such a way that the series $\sum x^n$ converges for every x with $\eta(x) < 1$. We show by examples that $\rho \neq \eta$ and $\eta \neq \beta$ in general. However, we obtain that η is exactly the maximum of ρ and β. We finally compare η to some known radii introduced by W. Zelazko [9] and studied

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