The (outer) automorphism group of a group extension

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Abstract

If $K \rightarrowtail G \twoheadrightarrow Q$ is a group extension, then any automorphism of G which sends K into itself, induces automorphisms respectively on K and on Q. This subgroup of automorphisms of G is denoted by $\mathrm{Aut}(G,K)$ and is called the automorphism group of the extension $K \rightarrowtail G \twoheadrightarrow Q$. After establishing an interesting group action of $\mathrm{Aut}(K) \times \mathrm{Aut}(Q)$ on the set $\mathcal{H}^2(Q,K)$ of all 2-cohomology classes of Q with coefficients in K, a full description of $\mathrm{Aut}(G,K)$ and $\mathrm{Out}(G,K) = \mathrm{Aut}(G,K)/_{\mathrm{Inn}(G)}$ is obtained in terms of various commutative diagrams. This picture is as general as possible, hence covering and further complementing similar ideas developed earlier by C. Wells ([5]), P. Conner & F. Raymond ([1]), D.J.S. Robinson ([3], [4]) and the author ([2]).

1 Notations and preliminaries

If G is a group and $x \in G$, then $\mu(x)$ is the inner automorphism determined by x (sending $y \in G$ to xyx^{-1}), $\mu(G)$ is known as the inner automorphism group $\mathrm{Inn}(G)$ and $\mathrm{Out}(G) = \mathrm{Aut}(G)/_{\mathrm{Inn}(G)}$ is called the outer automorphism group of G. Write $p : \mathrm{Aut}(G) \twoheadrightarrow \mathrm{Out}(G)$ for the natural projection. For a subset X in G, $C_G X$ denotes the centralizer and $N_G X$ is the normalizer of X in G. Let $Z(G)$ be the center of G.

In the sequel of this paper, aspects of group cohomology (with non-abelian coefficients) will be intensively used. Therefore, we review some basic facts of this theory and meanwhile fix additional notations and terminology.

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