

Synthetic Differential Geometry of Jet Bundles

Hirokazu Nishimura

Abstract

The theory of infinite jet bundles provides the very foundation for the geometric theory of nonlinear partial differential equations, but it is hard to say that orthodox differential geometry is an appropriate vehicle of the former theory, as has been exhorted by Vinogradov and his Russian school. We contend that synthetic differential geometry initiated by Lawvere is the veriest framework for the theory of infinite jet bundles. Kock (1980) gave a synthetic treatment of the theory of jet bundles, but his approach was restricted to formal manifolds and inherited clumsiness from the standard theory. This paper gives an alternative synthetic treatment of infinite jet bundles, in which the pinpointed notion of (nonlinear) connection will play a predominant role and no remainders of coordinates can be seen. Contact vector fields of finite type are completely determined in this new context.

0 Introduction

The notion of jet is a far-reaching generalization of that of tangent vector. The theory of jet bundles provides a good framework for the general theory of nonlinear partial differential equations as well as the calculus of variations, for which the reader is referred to Bocharov et al. (1999), Gamkrelidze (1991, Chapter 5) and Krasil'shchik, Lychagin and Vinogradov (1986) as well as Saunders (1989). All the same the standard theory of jet bundles appears clumsy mainly because of its heavy use of coordinate manipulations. Although the coordinate representation of jets bears

Received by the editors April 2000.

Communicated by Y. Félix.

1991 *Mathematics Subject Classification* : 58A03, 53C05, 58A20, 35A30, 35G20, 51K10.

Key words and phrases : synthetic differential geometry, jet bundle, connection, contact transformation, contact vector field, prolongation, preconnection, geometric theory of nonlinear partial differential equations, Cartan connection.