Corrigenda to "Polarities of Symplectic Quadrangles"

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Lemma 1.2 of [1] claims that the elements of the group Γ_{ℓ} of all automorphisms of the symplectic quadrangle W(F) that fix a line ℓ pointwise are induced by block matrices of the form

$$\begin{pmatrix} c\mathbf{1} & 0\\ X & c^{-1}\mathbf{1} \end{pmatrix}$$
, where $c \in F^{\times}, X \in F^{2 \times 2}$ such that $(X\mathbf{j})' = X\mathbf{j}$.

This is false if the ground field F contains non-squares. In fact, easy computations yield (contrary to what is stated in the paper) that Γ_{ℓ} is induced by the subgroup of GSp_4F consisting of all block matrices of the form

$$T(c,d,X) := \begin{pmatrix} c\mathbf{1} & 0 \\ X & d\mathbf{1} \end{pmatrix}, \text{ where } c,d \in F^{\times}, X \in F^{2\times 2} \text{ such that } (X\boldsymbol{j})' = X\boldsymbol{j}.$$

Scalar multiples do not affect the action on the quadrangle, and we obtain

Lemma. The elements of Γ_{ℓ} are represented by the elements of the subgroup

$$\left\{ T(c,1,X) \mid c \in F^{\times}, X \in F^{2 \times 2}, (X\boldsymbol{j})' = X\boldsymbol{j} \right\} \leq \mathrm{GSp}_4 F.$$

For each $c \in F^{\times}$, the element $T(c, c^{-1}, X)$ used above induces the same automorphism as $T(c^2, 1, X)$. For any field with more than two elements, the subgroup $\Sigma \leq \Gamma_{\ell}$ induced by $\{T(c^2, 1, X) \mid c \in F^{\times}, X \in F^{2 \times 2}, (Xj)' = Xj\}$ consists of all squares of elements of Γ_{ℓ} . If every element of F is a square (in particular, in the case where F has two elements), the groups Γ_{ℓ} and Σ coincide.

Lemma 1.5.3 of [1] has to be corrected: For |F| = 3, the group $E_{\ell} = \Sigma$ is the commutator subgroup of Γ_{ℓ} .

The vector space structure introduced in [1] 1.6 on E_{ℓ} only describes the action of scalars that are squares in F. Thus Lemma 1.7.3 gives $\dim_{F^{\phi}} E_{\ell}$ rather than $\dim_F E_{\ell}$. Nevertheless, the crucial observation that $\dim_F E_{\ell} = \dim_F E_p$ implies $F^{\phi} = F$ remains valid. After this observation, only perfect fields of characteristic 2 are considered, and the distinction between Σ and Γ_{ℓ} vanishes.

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References

 Stroppel, Markus: *Polarities of symplectic quadrangles*, Bull. Belg. Math. Soc. 10 (2003), 437–449.

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