Permanence properties of amenable, transitive and faithful actions (Erratum)*

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P. Fima correctly pointed out that, in the proof of the genericity of $\mathcal{O}_2$ in the proof of Proposition 4 appeared in [Moo], the permutation σ' is *a priori* not well defined. This can be easily corrected if we can assume the Følner sequences in question to be A-invariant. The following lemma allows us to make this assumption:

Lemma 1. *Let X be a G-set, Y be a H-set and A be a common finite subgroup of G and H such that the A-actions are free. Let $\{C_n\}_{n\geq 1}$ be a Følner sequence of $G \curvearrowright X$ and $\{D_n\}_{n\geq 1}$ be a Følner sequence of $H \curvearrowright Y$ such that $|C_n| = |D_n|$, $\forall n \geq 1$. Then there exist A-invariant Følner sequences $\{C'_n\}_{n\geq 1}$ for $G \curvearrowright X$ and $\{D'_n\}_{n\geq 1}$ for $H \curvearrowright Y$ such that $|C'_n| = |D'_n|$, $\forall n \geq 1$.*

Proof. First of all, remark that the set $\{AC_n\}_{n\geq 1}$ is a A-invariant Følner sequence of G. Indeed, for every $g \in G$, we have

$$\begin{aligned}\frac{|AC_n \bigtriangleup gAC_n|}{|AC_n|} &= \frac{|\cup_{a\in A}\, aC_n \bigtriangleup \cup_{b\in A} gbC_n|}{|AC_n|} \leq \frac{\left|\cup_{a,b\in A}\,(aC_n \bigtriangleup gbC_n)\right|}{|AC_n|} \\ &\leq \sum_{a,b\in A} \frac{|C_n \bigtriangleup a^{-1}gbC_n|}{|AC_n|} \xrightarrow[n\to\infty]{} 0.\end{aligned}$$

Since $\lim_{n\to\infty} \frac{|AC_n|}{|C_n|} = 1$, by passing to a subsequence if necessary, we can suppose that $|AD_n| \leq |AC_n| \leq (1+\frac{1}{n})|C_n|$, for all n. Since the A-actions are free, there exists an injection $f_n : AD_n \hookrightarrow AC_n$ which is A-equivariant. Let $D'_n := AD_n$ and $C'_n := f_n(AD_n)$. Then $C'_n \subseteq AC_n$ and clearly $\frac{|C'_n|}{|AC_n|} \leq 1$. Moreover $\frac{|C'_n|}{|AC_n|} \geq \frac{1}{1+\frac{1}{n}}$, so that $\lim_{n\to\infty} \frac{|C'_n|}{|AC_n|} = 1$.

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