# Siamese objects, and their relation to color graphs, association schemes and Steiner designs

Mikhail Klin*        Sven Reichard        Andrew Woldar

## 1   Introduction

Our motivation stems from the paper [9], in which the authors introduce a class of decompositions of complete graphs into certain strongly regular graphs which share a common spread. Analyzing their construction, we soon came to realize that these strongly regular graphs are $GQ(s,t)$-graphs, and thus, due to a famous result of A.E. Brouwer, deletion of the spread leads in each case to an antipodal distance regular graph which is an $(s+1)$-fold cover of the complete graph $K_{st+1}$. A relational structure resulting from these distance regular graphs, plus a certain stratification of the spread, yields an association scheme.

We call such objects Siamese association schemes. After recalling some preliminary notions in Section 2, we formulate an axiomatic system for such objects, as well as for some more general ones, and we analyze the resulting combinatorial structures.

We emphasize that the goal of this paper is to introduce the reader to the new area of *"Siamese objects"* in a clear and compelling way. Due to space limitations, we have sought to achieve a desirable balance between general discussion, paying close attention to computational detail, and the presentation of a few nice *ad hoc* models, including elements of proof elaborated on objects of small size. A more comprehensive treatment is thus postponed to [11] and [12]. We refer also to [19] as an important source of information. Finally, we acknowledge the indispensable role played in our investigations by the computer packages COCO [6], [7], GAP [20], Grape [21] and nauty [16].

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