Classical Subspaces of Symplectic Grassmannians

B. N. Cooperstein

1 Introduction and Basic Concepts

We assume the reader is familiar with the concepts of a *partial linear rank two incidence geometry* $\Gamma = (\mathcal{P}, \mathcal{L})$ (also called a point-line geometry) and the Lie incidence geometries. For the former we refer to articles in [B] and for the latter see the paper [Co].

The *collinearity graph* of Γ is the graph $(\mathcal{P}, \Delta)$ where Δ consists of all pairs of points belonging to a common line. For a point $x \in \mathcal{P}$ we will denote by $\Delta(x)$ the collection of all points collinear with x. For points $x, y \in \mathcal{P}$ and a positive integer t a *path of length* t from x to y is a sequence $x_0 = x, x_1, \ldots, x_t = y$ such that $\{x_i, x_{i+1}\} \in \Delta$ for each $i = 0, 1, \ldots, t-1$. The *distance* from x to y, denoted by $d(x, y)$ is defined to be the length of a shortest path from x to y if some path exists and otherwise is $+\infty$.

By a *subspace* of Γ we mean a subset S such that if $l \in \mathcal{L}$ and $l \cap S$ contains at least two points, then $l \subset S$. $(\mathcal{P}, \mathcal{L})$ is said to be *Gamma space* if, for every $x \in \mathcal{P}, \{x\} \cup \Delta(x)$ is a subspace. A subspace S is *singular* provided each pair of points in S is collinear, that is, S is a clique in the collinearity graph of Γ. For a Lie incidence geometry with respect to a "good node" every singular subspace, together with the lines it contains, is isomorphic to a projective space, see [Co]. Clearly the intersection of subspaces is a subspace and consequently it is natural to define the subspace generated by a subset X of $\mathcal{P}$, $\langle X \rangle_\Gamma$, to be the intersection of all subspaces of Γ which contain X. Note that if $(\mathcal{P}, \mathcal{L})$ is a Gamma space and X is a clique then $\langle X \rangle_\Gamma$ will be a singular subspace.