Searching for maximal partial ovoids and spreads in generalized quadrangles

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1 Introduction

A (finite) *generalized quadrangle* (GQ) of order (s, t) is an incidence structure $S = (P, B, I)$, in which P and B are disjoint (non-empty) sets of objects, called *points* and *lines* respectively, and for which I is a symmetric point-line *incidence relation* satisfying the following axioms:

(i) Each point is incident with $t + 1$ lines ($t \geq 1$) and two distinct points are incident with at most one line.

(ii) Each line is incident with $s+1$ points ($s \geq 1$) and two distinct lines are incident with at most one point.

(iii) If x is a point and L is a line not incident with x, then there is a unique pair $(y, M) \in P \times B$ for which $x\, I\, M\, I\, y\, I\, L$.

Interchanging points and lines in S yields a GQ of order (t, s), which is called the *dual* S^D of S. For the theory of generalized quadrangles, we refer to [12].

Here we will consider the classical generalized quadrangles (with q a power of a prime):

- The *quadrics* $Q(4, q)$ and $Q^-(5, q)$: let $Q = Q(4, q)$ (resp. $Q = Q^-(5, q)$) be a non-singular quadric of projective index 1 of the projective space $PG(4, q)$ (resp. $PG(5, q)$). Then the points of Q together with the lines on Q (subspaces of maximal dimension on Q) form a GQ of order $(s, t) = (q, q)$ (resp. (q, q^2)).

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