

Reduced product objects in model categories*

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For the seventieth birthday of Dieter Pumplün

Introduction

In his influential 1967 monograph *Homotopical Algebra*, D. Quillen [8] described an abstract approach to homotopy theory enabling analogous theories to be defined in categories other than the category of spaces and continuous maps. Although the starting point in the classical homotopy theory of spaces and maps is the equivalence relation (between maps) of *homotopy*, in Quillen's approach it is that of a *model category*, that is to say, a category \mathbf{C} , together with three distinguished classes of morphisms, *we*, *cof*, *fib*, called weak equivalences, cofibrations, and fibrations, respectively. These are required to satisfy certain axioms which reflect typical properties of the classes of such maps in topology and they enable the construction of much of the basic machinery of homotopy theory in the category \mathbf{C} . However, it is not possible in a model category to introduce all possible concepts and prove analogs of all possible theorems that hold in the homotopy theory of spaces: for the simple reason that the axioms of a model category are self-dual whereas the Eckmann-Hilton duality in spaces is known not to be perfect.

Nevertheless additional axioms, if enjoyed by a particular model category, sometimes enable further classical concepts and results to be introduced in \mathbf{C} . A relatively recent instance of this has been the successful definition by Doerane [2] of a notion of *Lusternik-Schnirelmann category* in a type of model category satisfying the so-called *cube axiom*. In such cases there is a price to be paid: the richer theory is only available in categories \mathbf{C} for which the additional axioms can be verified.

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