Solutions to the mean curvature equation by fixed point methods

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Abstract

We give conditions on the boundary data, in order to obtain at least one solution for the problem (1) below, with H a smooth function. Our motivation is a better understanding of the Plateau's problem for the prescribed mean curvature equation.

1 Introduction

We consider the Dirichlet problem in the unit disc $B = \{(u,v) \in \mathbf{R^2}; u^2 + v^2 < 1\}$ for a vector function $X : \overline{B} \longrightarrow \mathbf{R^3}$ which satisfies the equation of prescribed mean curvature

$$\begin{cases} \Delta X = 2H\,(X)\,X_u \wedge X_v \text{ in } B \\ \\ X = g \text{ on } \partial B \end{cases} \tag{1}$$

where $X_u = \dfrac{\partial X}{\partial u}$, $X_v = \dfrac{\partial X}{\partial v}$, $\wedge$ denotes the exterior product in $\mathbf{R^3}$ and $H : \mathbf{R^3} \longrightarrow \mathbf{R}$ is a given continuous function. For $H \equiv H_0 \in \mathbf{R}$ and g non constant with $0 < |H_0|\,\|g\|_\infty < 1$ there are two variational solutions ([1], [3]). For H near H_0 in certain cases there exist also two solutions to the Dirichlet problem ([2], [6]). For H far from H_0 , under appropriated conditions on g and H it is possible to obtain more than two solutions ([4]).

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