The p-adic Finite Fourier Transform and Theta Functions

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A polarization on an abelian variety A induces an isogeny between A and its dual variety $\hat{A}$. The kernel of this isogeny is a direct sum of two isomorphic subgroups. If A is an analytic torus over a non-archimedean valued field then it is possible to associate with each of these subgroups a basis for a corresponding space of theta functions, cf. [5], [6].
The relation between these bases is given by a finite Fourier transform. Similar results hold for complex abelian varieties, cf. [3].

The field k is algebraically closed and complete with respect to a non-archimedean absolute value. The residue field with respect to this absolute value is $\overline{k}$.

1 The finite Fourier transform

In this section we consider only finite abelian groups whose order is not divisible by $char(\overline{k})$.
For such a group A we denote by $\widehat{A}$ the group of k-characters of A, i.e. $\widehat{A} = Hom(A, k^*)$. The vector space of k valued functions on A is denoted as $V(A)$.

Lemma 1.1 *Let A_1 and A_2 be finite abelian groups. Then $(\widehat{A_1 \times A_2})$ is isomorphic with $\widehat{A_1} \times \widehat{A_2}$.*

Proof The map $\theta : \widehat{A_1} \times \widehat{A_2} \to \widehat{A_1 \times A_2}$, defined by $\theta(\chi, \tau)(a_1, a_2) = \chi(a_1).\tau(a_2)$ is an isomorphism. ∎

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