On prior distributions which give rise to a dominated Bayesian experiment

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Abstract

When the statistical experiment is dominated (i.e. when all the sampling distributions are absolutely continuous w.r.t. a σ-finite measure), all the probability measures on the parameter space are prior distributions which give rise to a dominated Bayesian experiment.
In this paper we shall consider the family $\mathbb{D}$ of prior distributions which give rise to a dominated Bayesian experiment (w.r.t. a fixed statistical experiment not necessarily dominated) and we shall think the set of all the probability measures on the parameter space endowed by the total variation metric d.
Then we shall illustrate the relationship between $d(\mu, \mathbb{D})$ (where μ is the prior distribution) and the probability to have sampling distributions absolutely continuous w.r.t. the predictive distribution.
Finally we shall study some properties of $\mathbb{D}$ in terms of convexity and extremality and we shall illustrate the relationship between $d(\mu, \mathbb{D})$ and the probability to have posteriors and prior mutually singular.

1 Introduction.

In this paper we shall consider the terminology used in [5]. Let $(S, \mathcal{S})$ (*sample space*) and $(A, \mathcal{A})$ (*parameter space*) be two Polish Spaces and denote by $\mathbb{P}(\mathcal{A})$ and by $\mathbb{P}(\mathcal{S})$ the sets of all the probability measures on $\mathcal{A}$ and $\mathcal{S}$ respectively.
Furthermore let $(P^a : a \in A)$ be a fixed family of probability measures on $\mathcal{S}$ (*sampling distributions*) such that $(a \mapsto P^a(X) : X \in \mathcal{S})$ are measurable mappings w.r.t. $\mathcal{A}$.

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