Frobenius Collineations in Finite Projective Planes

Johannes Ueberberg

1 Introduction

Given a finite field $F = GF(q^n)$ of order q^n it is well-known that the map $f : F \to F$, $f : x \mapsto x^q$ is a field automorphism of F of order n, called the *Frobenius automorphism.* If V is an n-dimensional vector space over the finite field $GF(q)$, then V can be considered as the vector space of the field $GF(q^n)$ over $GF(q)$. Therefore the Frobenius automorphism induces a linear map over $GF(q)$

$$\begin{aligned} R &: V \to V \\ R &: x \mapsto x^q \end{aligned}$$

of order n on V. It follows that R induces a projective collineation φ on the $(n-1)$-dimensional projective space $PG(n-1, q)$. We call φ and any projective collineation conjugate to φ a *Frobenius collineation.* In the present paper we shall study the case $n = 3$, that is, the Frobenius collineations of the projective plane $PG(2, q)$.

Let $P = PG(2, q^2)$. Then every Singer cycle σ (see Section 3) of P defines a partition $\mathcal{P}(\sigma)$ of the point set of P into pairwise disjoint Baer subplanes. These partitions are called *linear Baer partitions* or, equivalently, *Singer Baer partitions* [17]. If ϱ is a Frobenius collineation of P, then we define $\mathcal{E}_\varrho$ to be the set of Baer subplanes of P fixed by ϱ. It turns out that for $q \equiv 2 \mod 3$ we have $|\mathcal{P}(\sigma) \cap \mathcal{E}_\varrho| \in \{0, 1, 3\}$ with $|\mathcal{P}(\sigma) \cap \mathcal{E}_\varrho| = 3$ if and only if $\varrho \in N_G(<\sigma>)$, where $G = PGL_3(q^2)$ (see 3.5).

Received by the editors March 1996.
Communicated by J. Thas.
1991 *Mathematics Subject Classification* : 51E20, 51E24.
Key words and phrases : diagram geometry, Baer partitions, Frobenius collineations.