Very ample line bundles on blown - up projective varieties

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Abstract

Let X be the blowing - up of the smooth projective variety V. Here we study when a line bundle M on X is very ample and, if very ample, the k-very ampleness of the induced embedding of X.

Introduction.

In the last few years several mathematicians studied (from many points of view and with quite different aims and techniques) the following situation. Let $\pi : X \mapsto V$ be the blowing - up of the variety V at finitely many points $P_1,...,P_s$; study the geometric and cohomological properties of X. Many authors (see e.g. [1, 4, 6, 7, 8, 9] and the references quoted there) were interested in the projective embeddings of X, e.g. to determine the very ample line bundles on X. Every $M \in Pic(X)$ is of the form $M \simeq \pi^*(F) - \sum_{1 \leq i \leq s} a_i E_i$ with $F \in Pic(V)$, $E_i = \pi^{-1}(P_i)$ the exceptional divisors and a_i integers. The very ampleness of M is studied in terms of F and the integers a_i. The conditions on F and on the integers a_i are usually both numerical (often obvious necessary conditions) and "positivity properties" of F. Often we are interested in the very ampleness of a line bundle M whose associated $F \in Pic(V)$ is of the form $F \simeq L \otimes R$ with $L, R \in Pic(V)$. It seems both natural and technically very useful to split the conditions on $F \simeq L \otimes R$ into conditions on L and conditions on R. Our main result on this topic is the following one. In all this paper we will use the notations (without the assumptions) introduced in its statement (as the following one). If P is a smooth point of a variety T, aP (with $a > 0$, a integer) will