Measures with finite semi-variation

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Abstract

The purpose of this paper is to characterize the Banach spaces and the locally convex spaces E for which bounded additive measures or bounded σ-additive measures with values in $\mathcal{L}(E, F)$, the space of continuous linear maps from E into F, are of bounded semi-variation for any Banach space or locally convex space F.

This paper gives an answer to a problem posed by D.H. Tucker in [6].

1 Introduction and Definitions

Among the more interesting and useful properties of operators which are representable by integrals of scalar valued functions with respect to scalar valued measures is that such integers (operators) have certain weakened forms of continuity with respect to the integrals. The main examples are the dominated convergence theorem, the monotone convergence theorem and the bounded convergence theorem.

When one moves afield from the scalar valued cases, these results become at best questionable. The relationships which exist in the scalar case relating convergence in measure and convergence pointwise almost everywhere no longer obtain. Pathological examples abound. In [7], examples are given which show that neither type of convergence implies the other. Indeed an example is given of a scalar valued sequence which converges point-wise everywhere to one and in measure to zero. In [8] an example is given of a vector valued sequence on $[0, 1]$, the measure being ordinary Lebesgue measure, in which the functions converge in measure, but no subsequence converges almost everywhere. The pathology in the first example was due to the nature of the measure, that in the second was due to the nature of the range space for the functions involved.

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