About an integral operator preserving meromorphic starlike functions

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Abstract

Let $\mathcal{U} = \{z \in \mathbb{C} : |z| < 1\}$ be the unit disc in the complex plane.Let Σ_k be the class of meromorphic functions f in $\mathcal{U}$ having the form:

$$f(z) = \frac{1}{z} + \alpha_k z^k + \cdots, 0 < |z| < 1, k \geq 0$$

A function $f \in \Sigma = \Sigma_0$ is called starlike if

$$\mathrm{Re}\left[-\frac{zf'(z)}{f(z)}\right] > 0 \text{ in } \mathcal{U}$$

Let denote by Σ_k^* the class of starlike functions inΣ_k and by A_n the class of holomorphic functions g of the form:

$$g(z) = z + a_{n+1}z^{n+1} + \cdots, z \in \mathcal{U}, n \geq 1$$

With suitable conditions on $k, p \in \mathbb{N}$,on $c \in \mathbb{R}$,on $\gamma \in \mathbb{C}$ and on the function $g \in A_{k+1}$, the author shows that the integral operator
$L_{g,c,\gamma} : \Sigma \to \Sigma$ defined by:

$$K_{g,c}(f)(z) \equiv \frac{c}{g^{c+1}(z)} \int_0^z f(t)g^c(t)\mathrm{e}^{\gamma t^p} dt, z \in \mathcal{U}, f \in \Sigma$$

maps Σ_k^* into Σ_l^*, where $l = \min\{p-1, k\}$.

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