Rational tree relations

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Abstract

We investigate forests and relations on trees generated by grammars in which the non-terminals represent relations. This introduces some synchronization between productions. We show that these sets are also solutions of systems of equations, that they are described by rational expressions involving union, substitution and iterated substitution, and that they are preserved by residuals. We show that they are the images of k-copying descending transducers. Finally, we isolate a subset of these relations which is preserved by composition.

1 Introduction

The rational transductions over a free monoid are relations satisfying several good properties:

1. The identity I, the converse R^{-1} and the (associative) composition of rational transductions are again rational transductions.

2. The image of a rational language is again a rational language. Together with (1), this implies that the domain and range of a rational transduction are rational.

3. The membership relation $(x, y) \in R$ is decidable.