On the left linear Riemann problem in Clifford analysis

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Abstract

We consider a left-linear analogue to the classical Riemann problem:

$$\begin{aligned} D_a u &= 0 \text{ in } \mathbb{R}^n \setminus \Gamma \\ u^+ &= H(x)u^- + h(x) \text{ on } \Gamma \\ |u(x)| &= \mathcal{O}(|x|^{\frac{n}{2}-1}) \text{ as } |x| \to \infty. \end{aligned}$$

For this purpose, we state a Borel-Pompeiu formula for the disturbed Dirac operator $D_a = D + a$ with a paravector a and some functiontheoretical results. We reformulate the Riemann problem as an integral equation:

$$P_a u + HQ_a u = h \text{ on } \Gamma,$$

where $P_a = \frac{1}{2}(I + S_a)$ and $Q_a = I - P_a$. We demonstrate that the essential part of the singular integral operator S_a which is constructed by the aid of a fundamental solution of $D + a$ is just the singular integral operator S associated to D. In case S_a is simply S and $\Gamma = \mathbb{R}^{n-1}$, then under the assumptions

1. $H = \sum\limits_{\beta} H_\beta e_\beta$ and all H_β are real-valued, measurable and essentially bounded;
2. $(1 + H(x))\overline{(1 + H(x))}$ and $H(x)\bar{H}(x)$ are real numbers for all $x \in \mathbb{R}^{n-1}$;
3. the scalar part H_0 of H fulfils $H_0(x) > \varepsilon > 0$ for all $x \in \mathbb{R}^{n-1}$,

the Riemann problem is uniquely solvable in $L_{2,\mathcal{C}}(\mathbb{R}^{n-1})$ and the successive approximation

$$u_n := 2(1 + H)^{-1}h - (1 + H)^{-1}(1 - H)Su_{n-1}, \quad n = 1, 2, \ldots,$$

Received by the editors July 1995.
Communicated by R. Delanghe.
1991 *Mathematics Subject Classification* : 35F15; 47B.
Key words and phrases : Riemann problem, Singular Cauchy-type operators.