Parameter-dependent solutions of the classical Yang-Baxter equation on sl(n,C).

M. Cahen V. De Smedt

Abstract

For any integers n and m ($m \geq 4$) such that $n+m$ is odd we exhibit triangular solutions of the classical Yang-Baxter equation on $sl((n+1)(m+2), \mathbb{C})$ parametrized by points of a quotient of complex projective space $\mathbb{P}^{\lfloor \frac{n}{2} \rfloor}(\mathbb{C})$ by the action of the symmetric group $Sym(\lfloor \frac{n+1}{2} \rfloor)$ and we prove that no two of these solutions are isomorphic.

1 Introduction

The motivation for this work is to exhibit solutions of the classical Yang-Baxter equations depending on a large number of parameters, Such solutions lead, by a construction indicated by Drinfeld [1], to quantum groups. We hope that these parameter-dependent quantum groups may have interesting geometrical applications [3].

2 The classical Yang-Baxter equation.

Let $\mathcal{G}$ be a finite-dimensional Lie algebra over $\mathbb{K}$ ($= \mathbb{R}$ or $\mathbb{C}$); an element $R \in \wedge^2\mathcal{G}$ is said to be a solution of the classical Yang-Baxter equation iff

$$[R, R] = 0$$

where $[\ ,\ ] : \wedge^2\mathcal{G} \otimes \wedge^2\mathcal{G} \to \wedge^3\mathcal{G}$ is the Schouten bracket, defined on bivectors by

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