Spectral asymptotics and bifurcation for nonlinear multiparameter elliptic eigenvalue problems

Tetsutaro Shibata

Abstract

This paper is concerned with the nonlinear multiparameter elliptic eigenvalue problem

$$\begin{aligned} &u''(r) + \frac{N-1}{r}u'(r) + \mu u(r) - \sum_{i=1}^{k} \lambda_i f_i(u(r)) = 0, \ \ 0 < r < 1, \\ &u(r) > 0, \ \ 0 \le r < 1, \\ &u'(0) = 0, u(1) = 0, \end{aligned}$$

where $N \ge 1, k \in N$ and $\mu, \lambda_i \ge 0$ $(1 \le i \le k)$ are parameters. The aim of this paper is to study the asymptotic properties of eigencurve $\mu(\lambda, \alpha) = \mu(\lambda_1, \lambda_2, \cdots, \lambda_k, \alpha)$ with emphasis on the phenomenon of bifurcation from the first eigenvalue μ_1 of $-\triangle|_D$ and on gaining a clearer picture of the bifurcation diagram. Here, $\alpha > 0$ is a normalizing parameter of eigenfunction associated with $\mu(\lambda, \alpha)$. To this end, we shall establish asymptotic formulas of $\mu(\lambda, \alpha)$ as $|\lambda| \to \infty, 0$.

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