

Solving some problems of advanced analytical nature posed in the SIAM-review

Carl C. Grosjean

In this paper, three SIAM-Review problems selected from Vol. 34 (1992) are reconsidered and treated using methods according to my own vision on them.

1 On alternating double sums¹

Consider the functions $S(v)$ and $C(v)$ defined as the sums of two infinite double series :

$$S(v) = \sum_{m=0}^{+\infty} \sum_{n=1}^{+\infty} (-1)^{m+n} \frac{\sin(2v\sqrt{m^2+n^2})}{\sqrt{m^2+n^2}}, \quad (1.1)$$

$$C(v) = \sum_{m=0}^{+\infty} \sum_{n=1}^{+\infty} (-1)^{m+n} \frac{\cos(2v\sqrt{m^2+n^2})}{\sqrt{m^2+n^2}}, \quad (1.2)$$

whereby it is indifferent in which order of succession of m and n the summations are carried out on account of the symmetry of the summands with respect to m and n . Find closed expressions for $S(v)$ and $C(v)$ for arbitrary real v and try to deduce from them whether the conjectures

$$S(v) = -v/2 \text{ if } -\pi/\sqrt{2} < v < \pi/\sqrt{2}, \quad C(v) = 0 \text{ if } v = \pm 5/4, \quad (1.3)$$

based upon numerical calculations, hold or not.

¹problem posed by Malte Henkel (University of Geneva, Geneva, Switzerland) and R.A. Weston (University of Durham, UK) (Problem 92-11*)

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