On Two Combination Rules for 0-1-Sequences

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1 Statement of problems and results

Consider two players who compete with each other by successively taking part in a series of 'win-or-lose' games for one person, henceforth called trials. Two *termination rules* are taken into consideration:

(i) Stop as soon as one player has reached a prespecified number N of successful trials;

(ii) stop after the Nth trial.

The player having more successes at the time of termination will be declared the final winner; under rule (ii) there is of course also the possibility of a tie. For determining who, at any given time, will carry out the next trial, two *switching rules* are deliberated:

Rule 1. Always alternate between the players;

Rule 2. the current player continues if and only if he/she was successful in the last trial.

We first assume that the outcomes scored by the two players form two deterministic sequences of 1's (for successes) and 0's (for failures), say $\mathcal{U} = (U_1, U_2, \ldots)$ and $\mathcal{V} = (V_1, V_2, \ldots)$. To present the problem in a formal manner, let us take either U_1 or V_1 to start with. Call the resulting sequence $X^{(i)} = (X_1^{(i)}, X_2^{(i)}, \ldots)$ if switching rule $i \in \{1, 2\}$ is used. Let $M, N \in \mathbb{N}$ be arbitrary positive integers. We consider, for the two rules, the set of all pairs $(\mathcal{U}, \mathcal{V})$ of 0-1-sequences for which there are more 1's from the U-sequence than from the V-sequence among the first N components of $X^{(i)}$ for $i = 1, 2$, and also the set of all $(\mathcal{U}, \mathcal{V})$ for which there are N 1's from the U's before there are M 1's from the V's. The gist of this paper is to show that some surprisingly simple inclusions and even equalities hold between these sets. In

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