

On the fundamental double four-spiral semigroup

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Abstract

We give a new description of the fundamental double four-spiral semigroup.

The fundamental four-spiral semigroup Sp_4 and the fundamental double four-spiral semigroup DSp_4 were introduced in [1], [3], and [4]. These semigroups are interesting examples of fundamental regular semigroups, and are indispensable building blocks of bisimple, idempotent-generated regular semigroups. Their basic properties are recalled in parts 1 and 2 of this note.

In part 3 we give an alternate construction of DSp_4 in terms of the free semigroup on two generators, as a set of quadruples with a simple, bicyclic-like multiplication. This permits shorter proofs and easier access to the main properties of DSp_4 : descriptions of DSp_4/\mathcal{L} and DSp_4/\mathcal{R} (part 4); reduced form of the elements (part 5); and the property of congruences $\mathcal{C} \not\subseteq \mathcal{L}$ that DSp_4/\mathcal{C} is completely simple (part 6).

1. Recall that Sp_4 is the semigroup

$$Sp_4 \cong \langle a, b, c, d; a^2 = a, b^2 = b, c^2 = c, d^2 = d, \\ a = ba, b = ab, b = bc, c = cb, c = dc, d = cd, d = da \rangle$$

generated by four idempotents a, b, c, d such that $a \mathcal{R} b \mathcal{L} c \mathcal{R} d \leq_L a$. (We denote Green's left preorder $x \in S^1 y$ by $x \leq_L y$). It is shown in [3] that every element of Sp_4 can be written uniquely in reduced form

$$[c](ac)^m[a], \quad [d](bd)^n[b], \quad [c](ac)^m ad (bd)^n [b],$$

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