On the locally finite chain algebra of a proper homotopy type

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Abstract

In the classical paper [A-H] Adams-Hilton constructed a free chain algebra which is an important algebraic model of a simply connected homotopy type. We show that this chain algebra (endowed with an additional structure given by a "height function") yields actually an invariant of a proper homotopy type. For this we introduce the homotopy category of locally finite chain algebras without using the usual methods of pro-categories. As examples we consider the locally finite chain algebras of $\mathbb{R}^{n+1}$, $S^2 \times S^2 - \{point\}$, and $\mathbb{C}P_2 - \{point\}$.

1   Proper homotopy types of locally finite polyhedra.

Let Top be the category of topological spaces. A map $f : X \to Y$ is *proper* if both f is closed and the fibre $f^{-1}(y)$ is compact for each point $y \in Y$. Let $Topp$ be the subcategory of Top consisting of topological spaces and proper maps. The unit interval $I = [0,1] \subset \mathbb{R}$ yields the cylinder $IX = X \times I$ in Top and $Topp$ such that these categories are I-categories in the sense of [BAH;I §3], compare [BP;I.3.9] or [ADQ1]. Hence the homotopy categories $Top/\simeq$ and $Topp/\simeq$ are defined, and isomorphism types in these categories are homotopy types and proper homotopy types respectively. We are interested in new algebraic invariants of the proper homotopy type of a locally finite polyhedron. A *polyhedron* X is a topological space homeomorphic to a simplicial complex; if every vertex belongs to only finitely many simplices the polyhedron is *locally finite*, this is the case if and only if the space X is