Rank three geometries associated with $PSL(3,4)$

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1 Introduction

In [Bu86] F. BUEKENHOUT started a research program in order to find a unifying combinatorial approach to all finite simple groups, inspired by J. TITS' famous theory of buildings (see e.g. [Ti74]), that is to classify diagram geometries fulfilling various conditions for all of these groups. This attempt has recently led to several collections of geometries (see [BCD95a], [BCD95b], [BDL94] and [BDL95]) using CAYLEY (see [De94]). The present paper should be regarded as a part of this program. Although $PSL(3,4)$ is a group of Lie-type and therefore wellknown in this context, it is of great interest since there are many connections between this group and some sporadic groups, e.g. the large Mathieu groups because of the famous construction of the 5-(24,8,1) design for M_{24} from the projective plane of order four (see [Lü69]), and $PSL(3,4)$ is a large subgroup of M_{22}. The geometries in this paper are also of another interest because they are examples of *building-like* geometries in the sense of [BuPa95], which are e.g. extensions of (generalized) polygons by linear spaces and thick diagram geometries.

We construct three geometries of rank three admitting $PSL(3,4)$ as a flag-transitive automorphism group in an easy combinatorial way using objects like hyperovals, Baer subplanes and unitals in $PG(2,4)$. Also, we describe these geometries by giving their diagrams and we determine their full automorphism and correlation groups. Our main result is the following.