Plane representations of ovoids

David G. Glynn

Abstract

Various ways of representing an ovoid of $\mathrm{PG}(3, q)$, q even, in $\mathrm{PG}(2, q)$ are studied. These come from a kind of 'spread' of lines of $\mathrm{AG}(3, q)$, and involve five kinds of 'pencils' of ovals in the plane.

1 Introduction

An *ovoid* of $\mathrm{PG}(3, q)$ is defined to be a set of $q^2 + 1$ points, no three collinear, $(q > 2)$. In other words, it is a (q^2+1)-cap of $\mathrm{PG}(3, q)$. Barlotti [2] and Panella [16] showed that when q is odd all ovoids are elliptic quadrics. The even case is still not completely solved.

Any ovoid corresponds to an inversive (or Möbius) plane, by taking the structure of the points together with the non-tangent plane sections as the 'circles'. In Dembowski [3] we see that every inversive plane of even order q can be constructed from an ovoid of $\mathrm{PG}(3, q)$, whereas for q odd there might even exist inversive planes of non-prime-power orders.

Remark 1 *Henceforth we shall assume that* $q = 2^h$, $h \in \mathbb{Z}$, $h \geq 2$.

There are two infinite sequences of known ovoids: these are the elliptic quadrics, which occur for every prime-power q; and also the Suzuki-Tits ovoids, which occur for $q = 2^h$, h odd, $h \geq 3$. Glynn [4] has shown that any new ovoid would have a small group of central automorphisms, and that it is unlikely to contain any conics as plane sections, because then a whole sequence of perhaps non-isomorphic ovoids

Received by the editors October 1997.

Communicated by J. W. P. Hirschfeld.

1991 *Mathematics Subject Classification.* 51E20, 51E15, 05B25, 51B10, 51A30, 51A45.

Key words and phrases. inversive plane, ovoid, oval, conic, projective plane, pencil, spread, generalized quadrangle.