Homotopy pull backs, homotopy push outs and joins

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Abstract

We show here how the techniques based on homotopy pull backs and push outs lead to simple proofs for apparently difficult (known or unknown) results. They can be used not only in the category of topological spaces, but also in any Quillen's model category. Many of them rely on the two 'join theorems' we prove here. Further applications are the study of holonomy, or of the Lusternik-Schnirelmann category.

Many of the usual constructions in topology are nothing else but homotopy pull backs, homotopy push outs, or joins (which are a combination of the two formers). Loop spaces, suspensions, mapping cones, Ganea spaces, Whitehead's fat wedges, holonomy, for instance, involve such constructions.

After having defined in section 1 'homotopy pull backs' and 'homotopy push outs' in the general context of a Quillen's model category, we introduce in section 2 the 'join' and 'smash product' constructions and give their properties. Section 3 is quite central as it is devoted to state and prove the two 'join theorems' which are key theorems in the sequel. Section 4 gives applications of the join theorems. Some known difficult results as those of Ganea (4.3) or Marcum (4.1) appear here are easy consequences of the join theorems. In section 5 we go further with Ganea's 'fibre-cofibre' and Whitehead's 'fat wedge' constructions. At last, we study the holonomy of a join of fibration sequences in section 6. All this appears unified by the same kind of techniques that rely on the same small amount of axioms and basic properties.