Generalized divisor problem

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The authors study the generalized divisor problem by means of the residue theorem of Cauchy and the properties of the Riemann Zeta function.

By an elementary argument similar to that used by Dirichlet in the divisor problem, Landau [7] proved that if α and β are fixed positive numbers and if $\alpha \neq \beta$, then

$$\sum_{m^\alpha n^\beta \leq x} 1 = \zeta(\beta/\alpha)x^{1/\alpha} + \zeta(\alpha/\beta)x^{1/\beta} + O(x^{1/(\alpha+\beta)}).$$

In 1952 H.E. Richert by means of the theory of Exponents Pairs (developed by J.G. van der Korput and E. Phillips) improved the above O-term (see [8] or [4] pag. 221). In 1969 E. Krätzel studied the three-dimensional problem. Besides, M.Vogts (1981) and A. Ivić (1981) got some interesting results which generalize the work of P.G. Schmidt of 1968. In 1987 A.Ivić obtained Ω-results for $\int_1^T \Delta_k^2(a_1, \ldots, a_k, x)dx$ where $\Delta_k(a_1, \ldots, a_k, x)$ is the error-term of the summatory function of $d(a_1, \ldots, a_k, n)$. Moreover, he proved that his results hold for the function

$$\zeta^{q_1}(b_1 s)\zeta^{q_2}(b_2 s)\zeta^{q_3}(b_3 s)\ldots, \quad 1 \leq b_1 < b_2 < b_3 \ldots$$

q_j, b_j being some positive integers. In 1983 E.Krätzel studied the many-dimensional problem

$$\sum_{m_1^{b_1} \ldots m_n^{b_n} \leq x} m_1^{a_1} \ldots m_n^{a_n},$$

using the properties of the Riemann Zeta-function. M. Vogts (1985) also studied that problem but using elementary methods. Here, we will analyse some sums of the above kind by means of non-elementary methods.

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