On a generalized nonlinear equation of Schrödinger type.

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Abstract

Here is established the global existence of smooth solutions to a generalized nonlinear equation of Schrödinger type in the usual Sobolev spaces H^s and certain weighted Sobolev spaces by using Leray-Schauder fixed point theorem and delicate a priori estimates.

1   Introduction

In recent years, the initial (initial-boundary) value problem for the nonlinear Schrödinger (NLS) equation

$$i\partial_t u + u_{2x} + 2|u|^2 u = 0 \tag{1.1}$$

and its generalized forms have been widely studied in a lot of papers such as [3-12, 19, 20]. The high order nonlinear Schrödinger equation is

$$i\partial_t u + u_{4x} + 8|u|^2 u_{2x} + 2u^2 u_{2x}^* + 4|u_x|^2 u + 6u_x^2 u^* + 6|u|^4 u = 0 \tag{1.2}$$

which is the second equation in the Lax hierarchy of NLS equation [1,3,16,21,22].

NLS equations have been of great interest due to their occurrences as mathematical models in several scopes of physics and their implication in the development of solitons and inverse scattering transform theory [1, 2, 4, 14, 16, 17, 21, 22]. Particularly, it is found that the nonlinear Schrödinger equation (1.1) and the following