Bialgebra structures on a real semisimple Lie algebra.

Véronique Chloup

Abstract

We describe some results on the classification of bialgebra structures on a real semisimple Lie algebra. We first describe the possible Manin algebras (i.e. big algebra in the Manin triple) for such a bialgebra structure. We then determine all the bialgebra structures on a real semisimple Lie algebra for the nonzero standard modified Yang-Baxter equation. Finally we consider the case of a real simple Lie algebra the complexification of which is not simple and we give some partial results about the bialgebra structures for any nonzero modified Yang-Baxter equation.

1 Definitions and notations.

Our work is a continuation of a paper from M. Cahen, S. Gutt and J. Rawnsley [1]; we use the same notations as theirs which we now recall.

Definition 1. (cf[3]) A *Lie bialgebra* $(\mathfrak{g}, p)$ is a Lie algebra $\mathfrak{g}$ with a 1-cocycle $p : \mathfrak{g} \to \Lambda^2\mathfrak{g}$ (relative to the adjoint action) such that $p^* : \mathfrak{g}^* \times \mathfrak{g}^* \to \mathfrak{g}^*$ $(\xi, \eta) \to [\xi, \eta]$ *with*

$$\langle [\xi, \eta], X\rangle = \langle \xi \ \wedge \eta, p(X)\rangle$$

is a Lie bracket on $\mathfrak{g}^*$. One also denotes the bialgebra by $(\mathfrak{g}, \mathfrak{g}^*)$.
A Lie bialgebra $(\mathfrak{g}, p)$ is said to be *exact* if the 1-cocycle p is a coboundary, $p = \partial Q$, for $Q \in \Lambda^2\mathfrak{g}$.

This means that $\partial Q_X = [X, Q]$ and then the condition for $(\mathfrak{g}, \partial Q)$ to be a Lie bialgebra is that the bracket $[Q,Q]$ be invariant under the adjoint action in $\Lambda^3\mathfrak{g}$.

Received by the editors November 1994
Communicated by Y. Félix
AMS Mathematics Subject Classification : 53C15, 58F05.
Keywords : Bialgebras, Manin algebras, modified Yang-Baxter equation.