Adjoints, Multi–adjoints, Pluri–adjoints.

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Abstract

Starting from a family of categories $\mathcal{A}_p$, where p ranges over a set P of indices, and functors $G_p : \mathcal{A}_p \longrightarrow \mathcal{X}$, $p \in P$, where $\mathcal{X}$ is a category and each G_p has a left adjoint F_p, we construct a category $\mathcal{A}$ and a functor $G : \mathcal{A} \longrightarrow \mathcal{X}$ that has a multi-adjoint in the sense of Y. Diers, and then a category $\overline{\mathcal{A}}$ and a functor $\overline{G} : \overline{\mathcal{A}} \longrightarrow \mathcal{X}$ that has a pluri-adjoint in the sense of the authors. These constructions show, at least in this instance, how the transition from adjoints to multi-adjoints to pluri-adjoints is performed. The problem of the existence and the characterization of a general class of pluri-adjoints arising in this manner invites further study.

1. Let P be a set, and let $(\mathcal{A}_p)_{p\in P}$ be a family of small categories. Let $\mathcal{X}$ be a small category. Assume that, for each $p \in P$, there is given a functor $G_p : \mathcal{A}_p \longrightarrow \mathcal{X}$ that has a left adjoint $F_p : \mathcal{X} \longrightarrow \mathcal{A}_p$, $F_p \dashv G_p$. Let $\mathcal{A}$ be the disjoint union of the categories $\mathcal{A}_p$, $p \in P$, that is, the coproduct in **Cat** (the category of small categories and functors between them) of the categories $\mathcal{A}_p$, $p \in P$. Recall that an *object* of $\mathcal{A}$ is then a pair (A, p) with A an object of $\mathcal{A}_p$ and $p \in P$. A *morphism* in $\mathcal{A}$ between (A, p) and (B, q) is a morphism $A \longrightarrow B$ in $\mathcal{A}_p$ if $p = q$ (otherwise there are no morphisms $(A, p) \longrightarrow (B, q)$). Thus,

$$\mathcal{A}((A,p),(B,q)) = \begin{cases} \mathcal{A}_p(A,B) & \text{if } p = q, \\ \emptyset & \text{if } p \neq q. \end{cases}$$

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