

Smash Products for G -sets, Clifford Theory and Duality Theorems

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Introduction

The categorical approach to the theory of group graded rings has been successful in recent years and in particular the use of smash products has provided several new ideas, cf. [4] and later [3], [10], [11], ... Another tool, the use of which is prompted upon us by the problems connected to the consideration of induced modules for a G -graded ring R with respect to a non-normal subgroup H of G , is the G -set theory, cf. [11]. The latter considerations stem from a general Clifford theory for group graded rings initiated by E. Dade, cf. [6] and [7] and extended recently in [9] and [12] to almost complete generality as far as graded modules are being concerned. However the G -set graded modules remained to be studied and in view of the Clifford theory with respect to a non-normal subgroup and the interest of the generalized Hecke algebras appearing in this theory, cf. [12], it is worthwhile to develop in some detail a theory of smash products for G -sets and its use in Clifford theory and duality theorems as in [4], [2].

In Section 2 of this paper we let R be a G -graded ring and A a G -set; the main results in this section, Theorem 2.1. and Theorem 2.7. yield a category isomorphism between $R\#A$ -mod, that is the category of left modules over the smash product of R and A , and (G, A, R) -gr, that is the category of A -graded left modules over the G -graded ring R .

In Section 3 we extend E. Dade's Clifford theory to the case of G -set gradations; the main results in this section are Theorem 3.5. and Theorem 3.10. (the G -set

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