On the Kronecker Product of Schur Functions of Two Row Shapes

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Abstract

The Kronecker product of two homogeneous symmetric polynomials P_1 and P_2 is defined by means of the Frobenius map by the formula $P_1 \otimes P_2 = F(F^{-1}P_1)(F^{-1}P_2)$. When P_1 and P_2 are Schur functions s_λ and s_μ respectively, then the resulting product $s_\lambda \otimes s_\mu$ is the Frobenius characteristic of the tensor product of the irreducible representations of the symmetric group corresponding to the diagrams λ and μ. Taking the scalar product of $s_\lambda \otimes s_\mu$ with a third Schur function s_ν gives the so-called Kronecker coefficient $g_{\lambda\mu\nu} = \langle s_\lambda \otimes s_\mu, s_\nu \rangle$ which gives the multiplicity of the representation corresponding to ν in the tensor product. In this paper, we prove a number of results about the coefficients $g_{\lambda\mu\nu}$ when both λ and μ are partitions with only two parts, or partitions whose largest part is of size two. We derive an explicit formula for $g_{\lambda\mu\nu}$ and give its maximum value.

0 Introduction

Let $A(S_n)$ denote the group algebra of S_n, the symmetric group on n letters, i.e. $A(S_n) = \{f : S_n \rightarrow \mathbb{C}\}$ where $\mathbb{C}$ denotes the complex numbers. Let $C(S_n)$ denote the set of class functions of $A(S_n)$, i.e. those $f \in A(S_n)$ which are constant on

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