Grassmannian structures on manifolds

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Abstract

Grassmannian structures on manifolds are introduced as subbundles of the second order framebundle. The structure group is the isotropy group of a Grassmannian. It is shown that such a structure is the prolongation of a subbundle of the first order framebundle. A canonical normal connection is constructed from a Cartan connection on the bundle and a Grassmannian curvature tensor for the structure is derived.

1 Introduction

The theory of Cartan connections has lead S. Kobayashi and T. Nagano, in 1963, to present a rigourous construction of projective connections [3]. Their construction, relating the work of Eisenhart, Veblen, Thomas a.o. to the work of E. Cartan, has a universal character which we intend to use in the construction of Grassmannian-like structures on manifolds. The principal aim is to generalise Grassmannians in a similar way. By doing so we very closely follow their construction of a Cartan connection on a principal bundle subjected to curvature conditions and the derivation of a normal connection on the manifold.

The action of the projective group $Pl(n_o)$ on a Grassmannian $G(l_o, n_o)$ of l_o-planes in $\mathbb{R}^{n_o}$ is induced from the natural action of $Gl(n_o)$ on $\mathbb{R}^{n_o}$. Let H be the isotropy group of this action at a fixed point e of $G(l_o, n_o)$. The generalisation will consist in the construction of a bundle P with structure group H and base manifold

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