Quadric Representation and Clifford Minimal Hypersurfaces

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Abstract

The Clifford minimal hypersurfaces in the $(n+1)$-dimensional unit hypersphere are the only compact, minimal and non-totally geodesic spherical hypersurfaces possessing the following property: Its quadric representation is mass-symmetric in some hypersphere and minimal in some concentric hyperquadric.

1 Introduction

It is known that the class of minimal hypersurfaces in the $(n+1)$-dimensional unit hypersphere S^{n+1} is extremely large. The aim of this paper is to show that within this class those hypersurfaces possessing certain eigenvalue-behaviour of the products of coordinate functions are rigid. By a well known result of T. Takahashi [6] the coordinate functions of minimal hypersurfaces in S^{n+1} are eigenfunctions of the Laplace operator with the same eigenvalue. Consequently, in order to study the eigenvalue-behaviour of the products of coordinate functions it is natural to immerse the unit sphere by its second standard immersion.

In [5] A. Ros developed this idea in order to study compact, minimal submanifolds in S^{n+1}. He studied this problem by organizing the products of coordinate functions as a new isometric immersion into the space $SM(n+2)$ of $(n+2)\times(n+2)$ symmetric matrices, the so called quadric representation (for some details, see the next section). By using this idea M. Barros and O.J. Garay in [1] obtained a new characterization of Clifford torus among all compact, minimal and non-totally geodesic

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