Geometric hyperplanes of the half-spin geometries arise from embeddings

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Dedicated to J. A. Thas on his fiftieth birthday

Abstract

Let the point-line geometry $\Gamma = (\mathcal{P}, \mathcal{L})$ be a half-spin geometry of type $D_{n,n}$. Then, for every embedding of Γ in the projective space $\mathbb{P}(V)$, where V is a vector space of dimension 2^{n-1}, it is true that every hyperplane of Γ arises from that embedding. It follows that any embedding of this dimension is universal. There are no embeddings of higher dimension. A corollary of this result and the fact that Veldkamp lines exist ([6]), is that the Veldkamp space of any half-spin geometry ($n \geq 4$) is a projective space.

1 Introduction

Let $\Gamma = (\mathcal{P}, \mathcal{L})$ be a rank 2 incidence system, which we will call a *point-line* geometry. A *subspace* X is a subset of the set of points with the property that any line having at least two of its incident points in X, in fact has all its incident points in X. A proper subspace X is called a *geometric hyperplane* of Γ if and only if every line has at least one of its points in X.

Example. If $\mathbb{P} = \mathrm{PG}(n, F)$ is a projective space of (projective) dimension $n \geq 2$, truncated to its points and lines, then an ordinary projective hyperplane is a geometric hyperplane. (We shall often drop the adjective "geometric" and simply refer to geometric hyperplanes as "hyperplanes".)

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