Collineations of the Subiaco generalized quadrangles

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Dedicated to J. A. Thas on his fiftieth birthday

Abstract

Each generalized quadrangle (GQ) of order (q^2, q) derived in the standard way from a conical flock via a q-clan with $q = 2^e$ has subquadrangles of order q associated with a family of $q + 1$ (not necessarily projectively equivalent) ovals in $\mathrm{PG}(2, q)$. A new family of these GQ is announced in [1] and named the Subiaco GQ. We begin a study of their collineation groups. When e is odd, $e \geq 5$, the group is determined. In the standard notation for the GQ, the collineation group is transitive on the lines through the point (∞). As a corollary we have that up to the usual notions of equivalence, just one conical flock, one oval in $\mathrm{PG}(2, q)$, and one subquadrangle of order (q, q) arise.

1 Introduction

The objects studied in this paper are introduced in [1], and we thank its authors for making their work available to us as it was being developed. Moreover, Tim Penttila and Gordon Royle helped us eliminate a serious error in an early version of this work.

Let $F =$ GF(q), $q = 2^e$. For each $t \in F$, let $A_t = \begin{pmatrix} x_t & y_t \\ 0 & z_t \end{pmatrix}$ be a 2×2 matrix over F. Put $\mathcal{C} = \{A_t : t \in F\}$. Then $\mathcal{C}$ is a q-clan provided $A_t - A_s$ is anisotropic (i.e., $\alpha(A_t - A_s)\alpha^T = 0$ if and only if $\alpha = (0, 0)$) whenever $t, s \in F$, $t \neq s$. This holds if and only if $(x_t - x_s)(z_t - z_s)(y_t - y_s)^{-2}$ has trace 1 whenever $s \neq t$. From

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