ON 2PFA's AND THE HADAMARD QUOTIENT OF FORMAL POWER SERIES *

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Abstract

We present the main results on 2PFA's and on the Hadamard quotient of formal power series, the connection between the two notions being a result stating that the event defined by a 2PFA is the Hadamard quotient of two rational power series.

1   Introduction

A two-way probabilistic finite automaton (2PFA) is a machine consisting of a probabilistic finite-state control and an input tape which is scanned by a single two-way head, that is, the head can move both left and right.

The Hadamard quotient of rational power series φ and ψ is the power series associating with a string x the quotient $\varphi(x)/\psi(x)$, whenever defined.

With every 2PFA, it can be associated a power series called the *event*, that is the series associating to a string its probability to be accepted by the 2PFA. In the opposite to the one-way case, the event defined by a 2PFA is not always a rational power series. What turns out, and it is the motivation of this double subject paper, is that events defined by 2PFA's can be expressed in terms of rational power series; namely they are always the Hadamard quotient of two rational power series.

In Section 2 we present some classical results on the computational power of probabilistic automata pointing out the relationships between the two-way case and the one-way case.

Section 3 concerns the literature on the Hadamard quotient of power series. The most of the works on this subject deal with power series on one-letter alphabets and consider some analytical problems such as some partial solutions to the Pisot's conjecture, the conjecture asking whether the Hadamard quotient of two rational power series is still rational. On the contrary, in [1, 2] it is investigated the class $\mathcal{H}ad(K, \Sigma)$ which is the closure of rational power series by the Hadamard inverse, in the general case of a several letters alphabet.

Section 4 states the relation between 2PFA's and the Hadamard quotient, i.e. the result stating that the event defined by a 2PFA is always the Hadamard quotient of