A Basis for the non-Archimedian Holomorphic Theta Functions

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In this paper we introduce an analytic version of the concept of a theta group on a non-archimedean analytic torus.We construct a basis for a vector space of theta functions similar with the basis for the global sections of a ample line bundle such as given in [2].

Notations: k is a complete non-archimedean valued field. We assume k to be algebraically closed.

1 Analytic tori and 1-cocycles

Let $T = G/\Lambda$ be an analytic torus ; $G = (k^*)^g$ and $\Lambda \subset G$ is a lattice.
Let A be the group of nowhere vanishing holomorphic functions on G and let H be the character group of G. The lattice Λ acts on A in a canonical way: $\alpha^\gamma(x) = \alpha(\gamma x)$. Each 1-cocycle $\xi \in \mathcal{Z}^1(\Lambda, A)$ has a canonical decomposition of the following form:

$$\xi_\gamma(x) = c(\gamma).p(\gamma, \sigma(\gamma)).\sigma(\gamma)(x) \qquad \text{with}$$

(1) $c \in \mathrm{Hom}(\Lambda, k^*)$;
(2) $\sigma \in \mathrm{Hom}(\Lambda, H)$ such that $\sigma(\gamma)(\delta) = \sigma(\delta)(\gamma)$ for all $\gamma, \delta \in \Lambda$;
(3) $p : \Lambda \times H \to k^*$ a bihomomorphism such that $p(\gamma, u)^2 = u(\gamma)$ for all $\gamma \in \Lambda$ and $u \in H$.
(We may assume that $p(\gamma, \sigma(\delta)) = p(\delta, \sigma(\gamma))$ for all $\gamma, \delta \in \Lambda$.)

We will always assume that ξ is non-degenerate and positive i.e. σ is injective and $|\sigma(\gamma)(\gamma)| < 1$ for all $1 \neq \gamma \in \Lambda$. The existence of such a cocycle implies that T is an abelian variety, (see [1]).

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