Strong Barrelledness Properties in Lebesgue-Bochner Spaces

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Abstract

If (Ω, Σ, μ) is a finite atomless measure space and X is a normed space, we prove that the space $L_p(\mu, X)$, $1 \leq p \leq \infty$ is a barrelled space of class $\aleph_0$, regardless of the barrelledness of X. That enables us to obtain a localization theorem of certain mappings defined in $L_p(\mu, X)$.

By "space" we mean a "real or complex Hausdorff locally convex space". Given a dual pair (E, F), as usual $\sigma(E, F)$ denotes the weak topology on E. If B is a subset of a linear space E, $\langle B \rangle$ will denote its linear hull.

Let s be a positive integer, then a family $W = \{E_{m_1 m_2 \ldots m_p}, m_r \in \mathbb{N}, 1 \leq r \leq p \leq s\}$ of subspaces of E is said to be an s-net in E if $\{E_{m_1}, m_1 \in \mathbb{N}\}$ is an increasing covering of E and $\{E_{m_1 m_2 \ldots m_j}, m_j \in \mathbb{N}\}$ is an increasing covering of $E_{m_1 m_2 \ldots m_{j-1}}$, for $2 \leq j \leq s$.

A space E is suprabarrelled, or barrelled space of class 1, [17], if given an increasing covering of subspaces of E there is one of them which is dense and barrelled. For a given natural number $s \geq 2$, E is said to be barrelled of class s, [11] and [14], if given an increasing covering of subspaces of E there is one of them which is barrelled of class $s-1$. If E is barrelled of class s for each $s \in \mathbb{N}$ it is said that E is barrelled of class $\aleph_0$. It is shown in [11] that each non-normable Fréchet space has a dense subspace of class $s-1$ which is not barrelled of class s for all $s \geq 2$. On the other hand, if Σ denotes any σ-algebra of subsets of a set Ω, the space $\ell_0^\infty(\Omega, \Sigma)$ of all scalar Σ-simple functions defined on Ω with the supremum norm is a barrelled space of class $\aleph_0$, [9]. Since each barrelled space of class s is Baire-like, [15], it

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