The "statistical experiment"-equivalence for prior distributions

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Abstract

Two prior distributions are said to be $(P^a : a \in A)$-equivalent when they have in common all the families of posterior distributions (with respect to a fixed statistical experiment $(P^a : a \in A)$).
It is shown that two $(P^a : a \in A)$-equivalent prior distributions are necessarily mutually absolutely continuous and two cases of statistical experiment in some sense opposite are presented.
Furthermore a partial order for statistical experiments can be defined in a natural way by comparing the quotient sets of prior distributions w.r.t. the $(P^a : a \in A)$-equivalences.
Finally a result about the ϵ-contaminations is presented.

1 Introduction and preliminaries

In this paper we shall refer to the frame of *Bayesian experiments* (see e.g. [5]). Throughout this paper we shall denote the *parameter space* by $(A, \mathcal{A})$ and the *sample space* by $(S, \mathcal{S})$ and we shall assume they are two Polish spaces. Then, given a Markov kernel $(P^a : a \in A)$ from $(A, \mathcal{A})$ to $(S, \mathcal{S})$ and a probability measure μ on $\mathcal{A}$, we can consider the probability measure $\Pi_{\mu,(P^a:a\in A)}$ on $\mathcal{A} \otimes \mathcal{S}$ such that

$$\Pi_{\mu,(P^a:a\in A)}(E \times X) = \int_E P^a(X)d\mu(a), \quad \forall E \in \mathcal{A} \ \ and \ \ \forall X \in \mathcal{S}. \tag{1}$$

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