On UHL and HUL

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Abstract

Let R be a principal ideal domain of characteristic zero, containing $1/2$, and let $\varrho = \varrho(R) < \infty$ be the least non-invertible prime in R. Our main result is the following:

Let (L, d) be a connected differential non-negatively graded Lie algebra over R, whose underlying module is R-free of finite type. If $\mathrm{ad}^{\varrho-1}(x)(dx) = 0$, for homogeneous x in L_{even}, then the natural morphism $UFHL \to FHUL$ is an isomorphism of graded Hopf algebras; as usual, F stands for free part, H for homology, and U for universal enveloping algebra.

Related facts and examples are also considered.

This paper is a first part of a program of exploring the connections between UHL, the universal enveloping algebra of the homology of a differential graded Lie algebra L over a commutative ring containing $1/2$, and HUL, the homology of the universal enveloping algebra of L, via the natural morphism $UHL \to HUL$. For basic definitions, notation and results on the subject, we refer to standard references such as [2, 5, 9].

Our present goal is to prove the following:

1. Theorem. *Let R be a principal ideal domain of characteristic zero, containing $1/2$, and let $\varrho = \varrho(R) < \infty$ be the least non-invertible prime in R.*

Let further (L, d) be a differential graded Lie algebra over R, whose underlying module is R-free of finite type.

Let also L be r-reduced (i.e., L is trivial in dimensions less than r), with integer $r \geq 1$, and let $r' = 2[r/2 + 1]$ and non-negative integer $n < \varrho r' - 1$. Then: