# Weak convergence in spaces of measures and operators

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**Abstract**

J. K. Brooks and P. W. Lewis have established that if $E$ and $E^*$ have RNP, then in $M(\Sigma, E)$, $m_n$ converges weakly to $m$ if and only if $m_n(A)$ converges weakly to $m(A)$ for each $A \in \Sigma$. Assuming the existence of a special kind of lifting, N. Randrianantoanina and E. Saab have shown an analogous result if $E$ is a dual space. Here we show that for the space $M(\mathcal{P}(\mathbb{N}), E)$ where $E^*$ is a Grothendieck space or $E$ is a Mazur space, this kind of weak convergence is valid. Also some applications for subspaces of $L(E, F)$ similar to the results of N. Kalton and W. Ruess are given.

## 1 Introduction

Let $E$ and $F$ be two infinite dimensional Banach spaces. By $L(E, F)$ (resp. $K(E, F)$) we denote the Banach space of all bounded linear (resp. compact linear) operators from $E$ to $F$. The $\epsilon$-product $E\epsilon F$ is the operator space $K_{w^*}(E^*, F)$ of compact and weak$^*$-weak continuous linear operators from $E^*$ to $F$, endowed with the usual operator norm. Let $\Sigma$ be a $\sigma$-algebra on a non-empty set $S$, then $M(\Sigma, E)$ (resp. $ca(\Sigma, E)$) denotes the Banach space of all bounded countably additive vector measures endowed with the variation norm (resp. semivariation norm). The space $E$ is said to be Grothendieck if weak$^*$ and weak sequential convergence in $E^*$ coincide; $E$ is called Mazur if any weak$^*$-sequentially continuous linear functional on $E^*$ lies in $E$. For unexplained notations we refer the reader to [4], [5], [6].

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