Size homotopy groups for computation of natural size distances

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Abstract

For every manifold \mathcal{M} endowed with a structure described by a function from \mathcal{M} to the vector space \mathbb{R}^k , a parametric family of groups, called size homotopy groups, is introduced and studied. Some lower bounds for natural size distances are obtained in this way.

1 Introduction

Size Theory is a new approach to the problem of comparing manifolds endowed with a structure represented by an \mathbb{R}^k -valued function. Apart from the intrinsic and classical mathematical interest in this subject (think of the study of metric and conformal structures on manifolds), this theory has a strong motivation in Computer Vision for problems of Shape Recognition and Image Analysis and has turned out to be useful for several applications (see [4], [12], [13], [14], [15], [16] and [17]). On the other hand, mathematical problems arising in Computer Vision require new geometrical techniques (cf. [3], [11] and also the nice informal paper [2]).

In previous papers ([5], [6], [7], [8] and [9]) Size Theory was basically founded on two related concepts: natural size distances and size functions. Natural size distances are a tool for measuring the "difference" between two homeomorphic manifolds, on each of which a continuous \mathbb{R}^k -valued function, called *measuring function*,

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