On Small Congruence Covers

Dieter Jungnickel

Abstract

This note provides a group theoretic characterisation of small line covers of $PG(3,p)$ and, more generally, small congruence covers of $PG(2t+1,p)$. It is shown that any group of square order s^2 which admits a cover by at most $s+p-1$ subgroups of order s (where p is the smallest prime divisor of s) is necessarily elementary abelian; hence any such cover is in fact geometric, that is, a congruence cover of a suitable projective geometry. We also show that the preceding bound is essentially best possible: There exists a congruence cover with $s+p+1$ components in a suitable non-elementary abelian group whenever s is a proper power of a prime.

1 Introduction

Packing finite projective spaces with disjoint subspaces has for many years been a topic of considerable interest in Galois geometries. In particular, one studies *partial t-spreads*, that is, collections of pairwise disjoint t-dimensional subspaces in a space $PG(d,q)$. In spite of considerable effort, the fundamental question of determining the maximal size of a partial t-spread is still not settled in general; see Hirschfeld and Thas [12] for background. In contrast, the dual problem of *t-covers*, that is, minimal collections of t-dimensional subspaces covering $PG(d,q)$ is much better understood. The following result is due to Beutelspacher [3] (who determined the lower bound) and Eisfeld [7] (who gave the structural characterisation for the case of equality).

Result 1.1. *Let $\mathcal{C}$ be a t-cover of $PG(d,q)$, and write $d=a(t+1)+b$, where $0 \leq b \leq t$. Then*

$$|\mathcal{C}| \;\geq\; q^{b+1}(q^{(a-1)(t+1)}+\ldots+q^{2(t+1)}+q^{t+1}+1)+1,$$

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