Hypernormal Form Calculation for Triple-Zero Degeneracies

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Abstract

A computational approach to obtain normal forms for equilibrium points of three-dimensional autonomous systems, having a linear degeneracy corresponding to a triple-zero eigenvalue, is presented. Also, we provide the explicit expressions for the normal form coefficients, and analyze some additional simplifications that can be achieved.

The results are applied in the analysis of bifurcation behaviours in an autonomous electronic oscillator.

1 Introduction

The normal form theory is an useful tool to build, for the analysis of a given dynamical system, another one which is equivalent and easier to study. Typically, when one is dealing with a nonhyperbolic situation, the full consideration of nonlinear terms in the system is required. So, for each degeneracy in the linear part, it is very relevant to determine the nonlinear terms that can be removed by means of successive changes of variables, in order to obtain the simplest equivalent system which gives account of the original dynamics.

For the most frequent bifurcation cases, normal forms have been obtained (see [8], [10]). Here, following a line of previous works (see [7] and references therein) we will give a computational approach to build normal forms corresponding to a triple zero eigenvalue in the linear part. This situation was already considered in [10] but our approach seems to be more interesting from the point of view of applications: we give explicit expressions for the coefficients of the normal form and the changes of variables can be easily computed. Another important feature is that the algorithms used are very efficient when implemented in standard computer algebra systems.

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